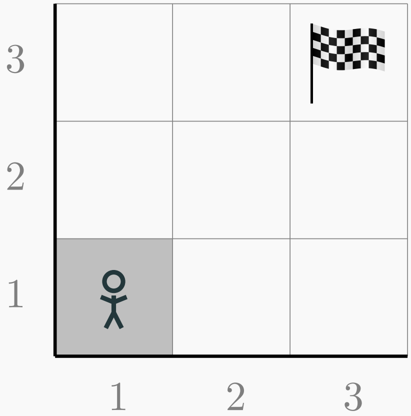
Potential Heuristics: Weakening Consistency Constraints

Pascal Lauer[1,2], Daniel Fišer[3]

[1] Saarland Informatics Campus, Germany
[2] Australian National University, Australia
[3] Aalborg University, Denmark

FDR Encoding Visitone



$$V = \{at\} \cup \{vis_{i,j} \mid (i,j) \in \{1,...,3\}^2\}$$

$$\text{dom}(at) = \{p_{i,j} \mid (i,j) \in \{1,...,3\}^2\}$$

$$\text{dom}(vis_{i,j}) = \{\top, \bot\}$$

$$O = \{move_{i_1,j_1,i_2,j_2} \mid (i_1,j_1,i_2,j_2) \in \{1,...,3\}^4$$
$$|i_1 - j_1| + |i_2 - j_2| = 1\}$$

$$c = \{o \mapsto 1 \mid o \in O\}$$

$$\mathcal{I} = \{at \leftarrow p_{1,1}, vis_{1,1} \leftarrow \top\}$$
$$\cup \{vis_{i,j} \leftarrow \bot \mid (i,j) \in \{1,...,n\}^2 \setminus \{(1,1)\}\}$$

$$\mathcal{G} = \{vis_{n,n} \leftarrow \top\}$$

$move_{i_1,j_1,i_2,j_2}$:

$pre : \{at \leftarrow p_{i_1,j_1}\}$

$eff : \{at \leftarrow p_{i_2,j_2}, vis_{i_2,j_2} \leftarrow \top\}$

Admissible Heuristics

Pro forma:

- $v \leftarrow x$ is a fact with $v \in \mathcal{V}, x \in \mathrm{dom}(v)$.
- A state is a set of facts.
- S is the set of states.

Admissible Heuristics

Pro forma:

- $v \leftarrow x$ is a fact with $v \in \mathcal{V}, x \in \mathrm{dom}(v)$.
- A state is a set of facts.
- S is the set of states.

A heuristic $h : S \mapsto \mathbb{R} \cup \{\infty\}$ is:

- **goal-aware**: $h(s) \leq 0$ for every $s \in S$ with $G \subseteq S$
- **consistent**: $h(s) \leq h(s') + c(o)$ for all $s \in S$ and s' reached by $o \in O$ applicable in s
- **admissible:** $h(s) \leq h^{\star}(s)$ for every $s \in S$

Admissible Heuristics

Pro forma:

- $v \leftarrow x$ is a fact with $v \in \mathcal{V}, x \in \mathrm{dom}(v)$.
- A state is a set of facts.
- S is the set of states.

A heuristic $h : S \mapsto \mathbb{R} \cup \{\infty\}$ is:

- **goal-aware**: $h(s) \leq 0$ for every $s \in S$ with $G \subseteq S$
- **consistent**: $h(s) \leq h(s') + c(o)$ for all $s \in S$ and s' reached by $o \in O$ applicable in s
- **admissible:** $h(s) \leq h^\star(s)$ for every $s \in S$
- Also: h is **admissible** if consistent and goal-aware

Admissible Heuristics

Pro forma:

- $v \leftarrow x$ is a fact with $v \in \mathcal{V}, x \in \mathrm{dom}(v)$.
- A state is a set of facts.
- S is the set of states.

A heuristic $h : S \mapsto \mathbb{R} \cup \{\infty\}$ is:

- **goal-aware**: $h(s) \leq 0$ for every $s \in S$ with $G \subseteq S$
- **consistent**: $h(s) \leq h(s') + c(o)$ for all $s \in S$ and s' reached by $o \in O$ applicable in s
- **admissible:** $h(s) \leq h^{\star}(s)$ for every $s \in S$
- Also: h is **admissible** if consistent and goal-aware

Strictly speaking the above definitions should consider forward reachable state and define each property p as "forward p".

(One Dimensional) Potential Heuristics

The potential heuristic $h^{\mathtt{P}} : S \rightarrow \mathbb{R} \cup \{\infty\}$ sums the potential values of a potential function $\mathtt{P} : \mathcal{F} \rightarrow \mathbb{R}$ over the facts in the given state s:

$$h^{\mathtt{P}}(s) = \sum_{f \in s} p(f)$$

Consistence Constraints for Opeator o

We ensure: “Potentials of facts deleted by o” - “Potentials of facts added by o” $\leq c(o)$

Consistence Constraints for Opeator o

We ensure: "Potentials of facts deleted by o" - "Potentials of facts added by o" $\leq c(o)$

If life was easy ($\mathcal{V}(\mathrm{eff}(o)) \subseteq \mathcal{V}(\mathrm{pre}(o))$):

$$\sum_{v \in \mathcal{V}(\mathrm{eff}(o)), v \leftarrow x \in \mathrm{pre}(o)} \mathtt{P}(v \leftarrow x) - \sum_{f \in \mathrm{eff}(o)} \mathtt{P}(f) \leq c(o)$$

$\mathcal{V}(\cdot)$ *denotes variables occuring in a set of facts.*

Consistence Constraints for Opeator o

We ensure: "Potentials of facts deleted by o" - "Potentials of facts added by o" $\leq c(o)$

If life was easy ($\mathcal{V}(\mathrm{eff}(o)) \subseteq \mathcal{V}(\mathrm{pre}(o))$):

$$\sum_{v \in \mathcal{V}(\mathrm{eff}(o)), v \leftarrow x \in \mathrm{pre}(o)} \mathtt{P}(v \leftarrow x) - \sum_{f \in \mathrm{eff}(o)} \mathtt{P}(f) \leq c(o)$$

$\mathcal{V}(\cdot)$ *denotes variables occuring in a set of facts.*

Extended (always correct):

$$\sum_{v \in \mathcal{V}(\mathrm{eff}(o)) \setminus \mathcal{V}(\mathrm{pre}(o))} \max_{x \in \mathrm{dom}(v)} \mathtt{P}(v \leftarrow x) + \sum_{v \in \mathcal{V}(\mathrm{eff}(o)), v \leftarrow x \in \mathrm{pre}(o)} \mathtt{P}(v \leftarrow x) - \sum_{f \in \mathrm{eff}(o)} \mathtt{P}(f) \leq c(o)$$

Disambiguations

$\mathcal{D}_o(v)$ denotes a subset of $\{v \leftarrow x \mid x \in \mathrm{dom}(v)\}$, restricted by the precondition of o. (Similarly, $\mathcal{D}_G(v)$ for the goal.)

Goal-Awareness:

$$\sum_{v \in \mathcal{V}} \max_{f \in \mathcal{D}_G(v)} \mathtt{P}(f) \leq 0$$

Consistency:

$$\sum_{v \in \mathcal{V}(\mathrm{eff}(o))} \max_{f \in \mathcal{D}_o(v)} \mathtt{P}(f) - \sum_{f \in \mathrm{eff}(o)} \mathtt{P}(f) \leq c(o)$$

Disambiguations

$\mathcal{D}_o(v)$ denotes a subset of $\{v \leftarrow x \mid x \in \mathrm{dom}(v)\}$, restricted by the precondition of o. (Similarly, $\mathcal{D}_G(v)$ for the goal.)

Goal-Awareness:

$$\sum_{v \in \mathcal{V}} \max_{f \in \mathcal{D}_G(v)} \mathtt{P}(f) \leq 0$$

Consistency:

$$\sum_{v \in \mathcal{V}(\mathrm{eff}(o))} \max_{f \in \mathcal{D}_o(v)} \mathtt{P}(f) - \sum_{f \in \mathrm{eff}(o)} \mathtt{P}(f) \leq c(o)$$

Besides the concise notation, they make an important difference for experiments.

$\mathtt{P}$ for (Almost) Perfect Heuristic

$at \leftarrow p_{x,y}$

2	1	0
3	2	1
4	3	2

(Manhattan Distance)

$vis_{x,y} \leftarrow \top$

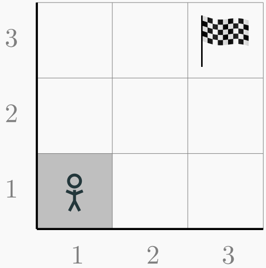
0	0	-4
0	0	0
0	0	0

(Cancel Out Positions)

$h^P(s) = h^*(s)$ *for all states* s *with* $h^*(s) > 0$. *Generalizes to* $n \times n$.

Investigating the heuristic

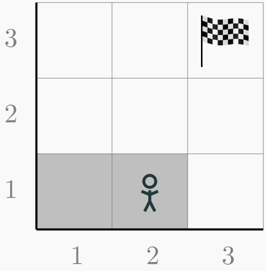
s :



$$h^{\mathsf{P}}(s) = 4$$

Investigating the heuristic

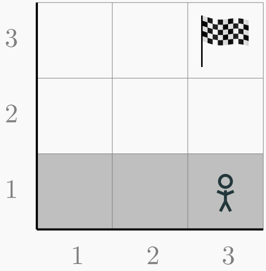
s :



$$h^{\mathsf{P}}(s) = 3$$

Investigating the heuristic

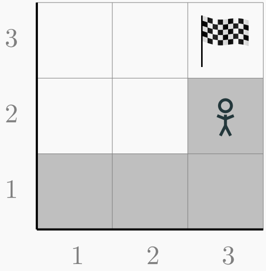
s :



$h^{\mathsf{P}}(s) = 2$

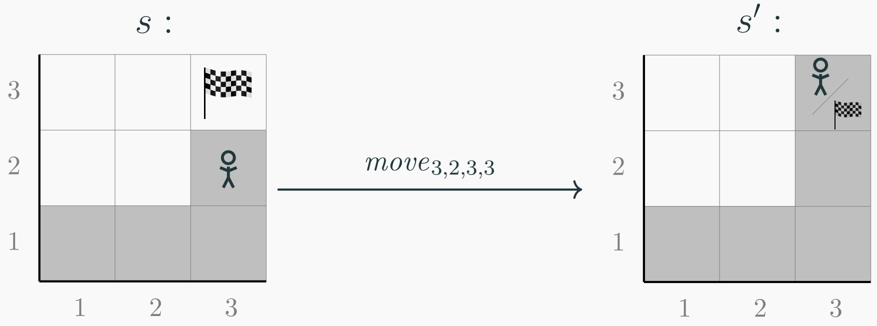
Investigating the heuristic

s :



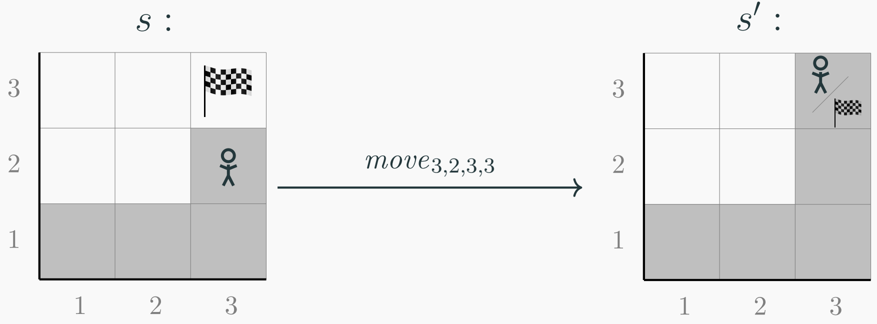
$$h^{\mathsf{P}}(s) = 1$$

Investigating the heuristic: It is not consistent!



$h^{\mathsf{P}}(s) = 1$ $\qquad$ $h^{\mathsf{P}}(s') = -4$

Investigating the heuristic: It is not consistent!



$h^{\mathsf{P}}(s) = 1$

$h^{\mathsf{P}}(s') = -4$

$$h^{\mathsf{P}}(s) - h^{\mathsf{P}}(s') = 1 - (-4) = 5 \not\leq 1 = c(move_{3,2,3,3})$$

Just to confirm: It violates our LP constraints

Consider the consistency constraint of h^{P} for $move_{3,2,3,3}$:

$$
\begin{aligned}
& p(at \leftarrow p_{3,2}) + \max_{x \in \{\top, \bot\}} p(vis_{3,3} \leftarrow x) - p(at \leftarrow p_{3,3}) - p(vis_{3,3} \leftarrow \top)) \\
= \quad & 1 \quad + \quad 0 \quad - \quad 0 \quad - \quad (-4) \quad = \quad 5 \\
\not\leq \; & 1 = c(move_{3,2,3,3})
\end{aligned}
$$

Just to confirm: It violates our LP constraints

Consider the consistency constraint of h^{P} for $move_{3,2,3,3}$:

$$\begin{aligned} & p(at \leftarrow p_{3,2}) + \max_{x \in \{\top,\bot\}} p(vis_{3,3} \leftarrow x) - p(at \leftarrow p_{3,3}) - p(vis_{3,3} \leftarrow \top)) \\ = \quad & 1 \quad + \quad 0 \quad - \quad 0 \quad - \quad (-4) \quad = \quad 5 \\ \not\leq \; & 1 = c(move_{3,2,3,3}) \end{aligned}$$

In fact one can prove any heuristic value to be bounded by ≤ 2 under our constraint encoding.

When is this a problem?

Intuitively prevents us from capturing distances of any kind. We can, e.g., also observe this in:

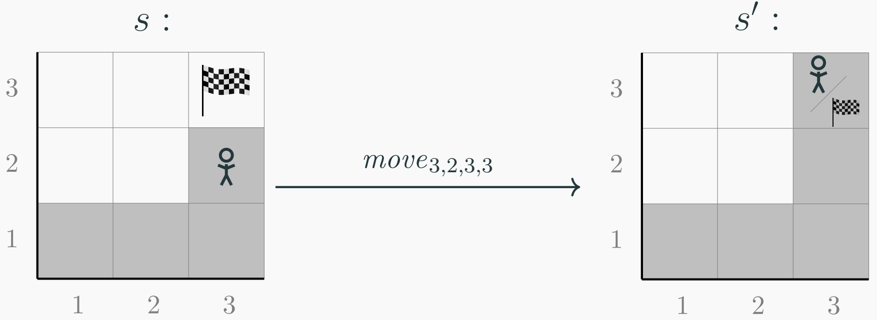
- Logistics (In the paper)
- Elevators (In the experiments)

But do we use an inconsistent heuristic (in search)?

$$h^{\mathtt{P}}_{0+}(s) := \max(h^{\mathtt{P}}(s), 0)$$

But do we use an inconsistent heuristic (in search)?

$$h^{\mathsf{P}}_{0+}(s) := \max(h^{\mathsf{P}}(s), 0)$$

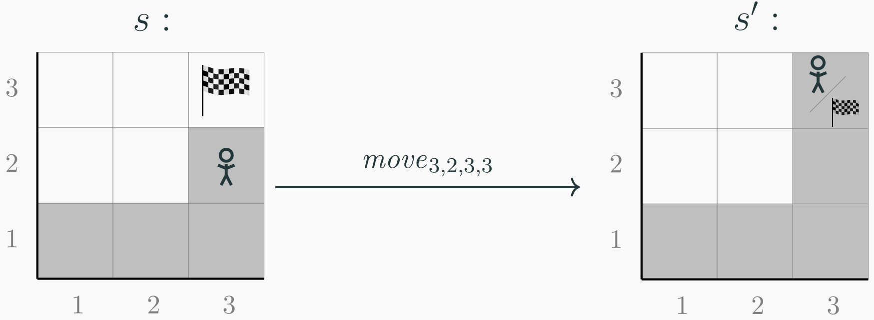


$h^{\mathsf{P}}_{0+}(s) = \max(0, 1) = 1$

$h^{\mathsf{P}}_{0+}(s) = \max(0, -4) = 0$

But do we use an inconsistent heuristic (in search)?

$$h^{\mathsf{P}}_{0+}(s) := \max(h^{\mathsf{P}}(s), 0)$$



$h^{\mathsf{P}}_{0+}(s) = \max(0, 1) = 1$

$h^{\mathsf{P}}_{0+}(s) = \max(0, -4) = 0$

$$h^{\mathsf{P}}_{0+}(s) - h^{\mathsf{P}}_{0+}(s') = 1 - 0 = 1 \leq c(move_{3,2,3,3})$$

Deriving consistency constraints for $h^{\mathtt{P}}_{0+}$

Consistency for $h^{\mathtt{P}}_{0+}$ in transition from s to s' via operator o:

$$\begin{aligned} h^{\mathtt{P}}_{0+}(s) - h^{\mathtt{P}}_{0+}(s') &\leq c(o) \\ \Leftrightarrow \max(h^{\mathtt{P}}(s), 0) - \max(h^{\mathtt{P}}(s'), 0) &\leq c(o) \end{aligned}$$

Deriving consistency constraints for $h^{\mathtt{P}}_{0+}$

Consistency for $h^{\mathtt{P}}_{0+}$ in transition from s to s' via operator o:

$$
\begin{aligned}
h^{\mathtt{P}}_{0+}(s) - h^{\mathtt{P}}_{0+}(s') &\leq c(o) \\
\Leftrightarrow \max(h^{\mathtt{P}}(s), 0) - \max(h^{\mathtt{P}}(s'), 0) &\leq c(o)
\end{aligned}
$$

$\Leftrightarrow$ do the math

$$\min(h^{\mathtt{P}}(s) - h^{\mathtt{P}}(s'), h^{\mathtt{P}}(s)) \leq c(o)$$

Deriving consistency constraints for $h^{\mathtt{P}}_{0+}$

Consistency for $h^{\mathtt{P}}_{0+}$ in transition from s to s' via operator o:

$$\begin{aligned} h^{\mathtt{P}}_{0+}(s) - h^{\mathtt{P}}_{0+}(s') &\leq c(o) \\ \Leftrightarrow \max(h^{\mathtt{P}}(s), 0) - \max(h^{\mathtt{P}}(s'), 0) &\leq c(o) \end{aligned}$$

$\Leftrightarrow$ **do the math**

$$\min(\underbrace{h^{\mathtt{P}}(s) - h^{\mathtt{P}}(s')}_{\texttt{old constraint}}, \underbrace{h^{\mathtt{P}}(s)}_{?}) \leq c(o)$$

Estimating $h^{\mathtt{P}}$s

Estimate on $h^{\mathtt{P}}(s)$ for all s where the precondition of o is fulfilled:

$$\sum_{v \in \mathcal{V}} \max_{f \in \mathcal{D}_o(v)} \mathtt{P}(f)$$

Estimating $h^{\mathtt{P}}$s

Estimate on $h^{\mathtt{P}}(s)$ for all s where the precondition of o is fulfilled:

$$\sum_{v \in \mathcal{V}} \max_{f \in \mathcal{D}_o(v)} \mathtt{P}(f)$$

Revisiting the example:

$$\begin{aligned} & p(at \leftarrow p_{3,2}) + \sum_{(i,j) \in \{1,..,3\}^2} \max_{x \in \{\top,\bot\}} p(vis_{i,j} \leftarrow x) \\ = & \quad 1 \quad + \quad 0 \\ \leq & \ 1 = c(move_{3,2,3,3}) \end{aligned}$$

Putting it together with indicator constraints

$$\min(h^{\mathsf{P}}(s) - h^{\mathsf{P}}(s'), h^{\mathsf{P}}(s)) \leq c(o)$$

Putting it together with indicator constraints

$$\min(h^{\mathtt{P}}(s) - h^{\mathtt{P}}(s'), h^{\mathtt{P}}(s)) \leq c(o)$$

We want to encode an or:

$$ind = 0 \Rightarrow \sum_{v \in \mathcal{V}(\text{eff}(o))} \max_{f \in \mathcal{D}_o(v)} \mathtt{P}(f) - \sum_{f \in \text{eff}(o)} \mathtt{P}(f) \leq c(o)$$

$$ind = 1 \Rightarrow \quad \sum_{v \in \mathcal{V}} \max_{f \in \mathcal{D}_o(v)} \mathtt{P}(f)$$

Putting it together with indicator constraints

$$\min(h^{\texttt{P}}(s) - h^{\texttt{P}}(s'), h^{\texttt{P}}(s)) \leq c(o)$$

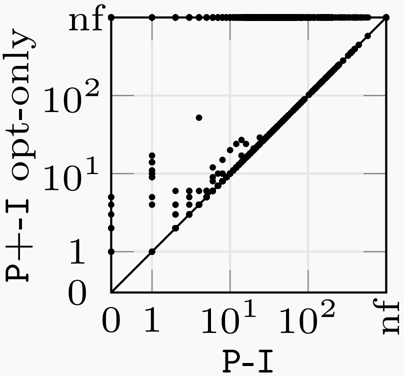
We want to encode an or:

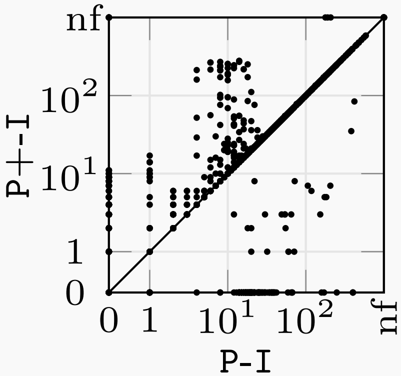
$$ind = 0 \Rightarrow \sum_{v \in \mathcal{V}(\text{eff}(o))} \max_{f \in \mathcal{D}_o(v)} \texttt{P}(f) - \sum_{f \in \text{eff}(o)} \texttt{P}(f) \leq c(o)$$

$$ind = 1 \Rightarrow \quad \sum_{v \in \mathcal{V}} \max_{f \in \mathcal{D}_o(v)} \texttt{P}(f)$$

Pro: Optimal solution will be stricty better

Con: We need an ILP





P+ is our ILP variant. opt-only requires to to find an optimal solution. (Other configuration takes best after timeout.)

Experiments (Coverage)

Domain	I			A			AI		
	P	P+	por	P	P+	por	P	P+	por
blocks (35)	21	**28**	**28**	28	28	28	28	28	28
caldera (20)	10	**12**	10	12	12	12	12	12	12
driverlog (20)	9	**13**	**13**	**13**	11	**13**	13	13	13
elevators (50)	31	31	31	31	**35**	**35**	31	**33**	**33**
freecell (80)	48	**60**	**60**	37	37	37	**74**	69	**74**
ged (20)	16	**18**	**18**	15	15	15	15	**18**	**18**
logistics (63)	13	**23**	**23**	24	24	24	24	24	24
mprime (35)	18	**20**	19	25	25	25	25	25	25
nomystery (20)	10	**14**	**14**	**14**	13	**14**	14	14	14
parking (40)	1	**7**	**7**	**16**	6	**16**	**16**	7	**16**
pipesw-notank (50)	25	**26**	**26**	25	25	25	**30**	28	**30**
recharging-robots (20)	**13**	12	**13**	**13**	12	**13**	**13**	11	**13**
scanalyzer (50)	25	25	25	23	23	23	**27**	25	**27**
snake (20)	**15**	14	**15**	**14**	13	**14**	**16**	13	**16**
spider (20)	**14**	11	**14**	**14**	11	**14**	**16**	12	**16**
tidybot (40)	**32**	30	**32**	**32**	30	**32**	**32**	30	**32**
tpp (30)	6	**8**	**8**	6	6	6	8	8	8
trucks (30)	9	**14**	**14**	**14**	11	**14**	14	14	14
visitall (40)	25	**30**	**30**	25	**28**	**28**	27	**30**	**30**
woodworking (50)	19	**27**	**27**	**27**	25	**27**	31	31	31
zenotravel (20)	8	**11**	**11**	11	11	11	11	11	11
others (1053)	558	**566**	565	569	566	**570**	**584**	582	**584**
Σ (1806)	926	1 000	**1 003**	988	967	**996**	1 061	1 038	**1 069**

Thank you :)