

AI Planning

16. Combining Heuristic Functions Part II - LP Heuristics

“Beyond Optimal Cost Partitioning”

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Agenda

- 1 Introduction
- 2 Post-hoc Optimization Heuristic
- 3 Operator-counting Framework
- 4 Potential Heuristics
- 5 Conclusion

Our Agenda for This Chapter

- 2 **Post-hoc optimization:** Combining heuristics with a subset of relevant actions.
- 3 **Operator Counting Heuristics:** Interpreting heuristics as constraints on the set of actions that must be applied.
- 4 **Potential Heuristics:** Deriving heuristics via linear programming.

Combining Estimates from Abstraction Heuristics

We already know two approaches to derive heuristic estimates from a collection of abstractions:

(Reminder **Chapter 12**): Canonical Heuristic Function

$$h^C(s) = \max_{D \in \text{cliques}(\mathcal{C})} \sum_{P \in D} h^P(s).$$

(Reminder **Chapter 16**): Optimal Cost Partitioning: uses a **state-specific LP** to find the **best possible cost partitioning**, and sums up the heuristic estimates.

→ is **very expensive** to compute (recomputing the PDBs in every state).

Is there a way to combine multiple abstraction heuristics that **does not need to re-compute the distances** in every state and is **more informative than the canonical heuristic**?

Example Task (1)

Example (Example Task)

FDR task $\Pi = \langle V, I, O, \gamma \rangle$ with

- $V = \{A, B, C\}$ with $\text{dom}(v) = \{0, 1, 2, 3, 4\}$ for all $v \in V$
- $I = \{A \mapsto 0, B \mapsto 0, C \mapsto 0\}$
- $A = \{inc_x^v \mid v \in V, x \in \{0, 1, 2\}\} \cup \{jump^v \mid v \in V\}$
 - $inc_x^v = \langle v = x, v := x + 1, 1 \rangle$
 - $jump^v = \langle \bigwedge_{v' \in V: v' \neq v} v' = 4, v := 3, 1 \rangle$
- $\gamma = A = 3 \wedge B = 3 \wedge C = 3$

- Each optimal plan consists of three increment actions for each variable $\rightsquigarrow h^*(I) = 9$
- Each action affects only one variable.

Example Task (2)

- In projections on single variables we can reach the goal with a *jump* action: $h^{\{A\}}(I) = h^{\{B\}}(I) = h^{\{C\}}(I) = 1$.
- In projections on more variables, we need for each variable three applications of increment actions to reach the abstract goal from the abstract initial state: $h^{\{A,B\}}(I) = h^{\{A,C\}}(I) = h^{\{B,C\}}(I) = 6$

Example (Canonical Heuristic)

$\mathcal{C} = \{\{A\}, \{B\}, \{C\}, \{A, B\}, \{A, C\}, \{B, C\}\}$

$$h^{\mathcal{C}}(s) = \max\{h^{\{A\}}(s) + h^{\{B\}}(s) + h^{\{C\}}(s), h^{\{A\}}(s) + h^{\{B,C\}}(s), h^{\{B\}}(s) + h^{\{A,C\}}(s), h^{\{C\}}(s) + h^{\{A,B\}}(s)\}$$

$$h^{\mathcal{C}}(I) = 7$$

Post-hoc Optimization Heuristic: Idea

Consider the example task:

- **type- v action**: action modifying variable v
- $h^{\{A,B\}} = 6$
 \Rightarrow any plan contains **at least 6 actions of type A or B** .
- $h^{\{A,C\}} = 6$
 \Rightarrow any plan contains **at least 6 actions of type A or C** .
- $h^{\{B,C\}} = 6$
 \Rightarrow any plan contains **at least 6 actions of type B or C** .
- \Rightarrow **at least 9 actions** in any plan

Can we generalize this kind of reasoning?

Post-hoc Optimization Heuristic: Linear Program

For pattern collection \mathcal{C} :

Variables

X_a for each action $a \in A$

Objective

Minimize $\sum_{a \in A} X_a$

Subject to

$$\sum_{a \in A: a \text{ affects } P} X_a \geq h^P(s) \quad \text{for all patterns } P \in \mathcal{C}$$

$$X_a \geq 0 \quad \text{for all } a \in A$$

\rightarrow For each new state, just change the bounds $h^P(s)$.

Post-hoc Optimization Heuristic: Admissibility

Theorem (Admissibility): The post-hoc optimization heuristic is **admissible**.

Proof: Let Π be a planning task and \mathcal{C} be a pattern collection. Let π be an optimal plan for state s and let $c_\pi(A')$ be the cost incurred by actions from $A' \subseteq A$ in π .

Setting each $X_{[a]}$ to $c_\pi([a])$ is a feasible variable assignment: Constraints $X_{[a]} \geq 0$ are satisfied. For each $P \in \mathcal{C}$, π is a solution in the abstract transition system and the sum in the corresponding constraint equals the cost of the “true” abstract state transitions (i.e., not accounting for self-loops). As $h^P(s)$ corresponds to the cost of an optimal solution in the abstraction, the inequality holds.

For this assignment the objective function has value $h^*(s)$ (cost of π), so the objective value of the LP is admissible.

Relation to Canonical Heuristic

Theorem: The post-hoc optimization heuristic h_c^{PhO} dominates the canonical heuristic h^c for the same pattern collection \mathcal{C} .

Proof: Consider the dual D of the LP solved by h_c^{PhO} in state s for a given pattern collection \mathcal{C} : Maximize $\sum_{P \in \mathcal{C}} Y_P h^P(s)$ subject to

$$\sum_{P \in \mathcal{C}: o \text{ affects } P} Y_P \leq 1 \quad \text{for all } [o] \in O/\sim$$

$$Y_P \geq 0 \quad \text{for all } P \in \mathcal{C}.$$

We compute a state-specific cost partitioning that can only scale the action costs within each heuristic by a factor Y_i .

If we restrict the variables in D to integers, the objective value is the canonical heuristic value $h^c(s)$.

→For the canonical heuristic, we need to find all maximal cliques, which is an NP-hard problem.

→The post-hoc optimization heuristic dominates the canonical heuristic and can be computed in polynomial time.

Reminder: Optimal Cost Partitioning for Landmarks

Variables

Count_a for each action a

Objective

Minimize $\sum_a \text{Count}_a \cdot c(a)$

Subject to

$$\sum_{a \in L} \text{Count}_a \geq 1 \text{ for all landmarks } L$$

$$\text{Count}_a \geq 0 \text{ for all actions } a$$

Numbers of action occurrences in any plan satisfy constraints.

Minimizing the total plan cost gives an admissible estimate.

Can we apply this idea more generally?

Operator Counting

Operator-counting Constraints

- linear constraints whose variables denote number of occurrences of a given action
- must be satisfied by every plan

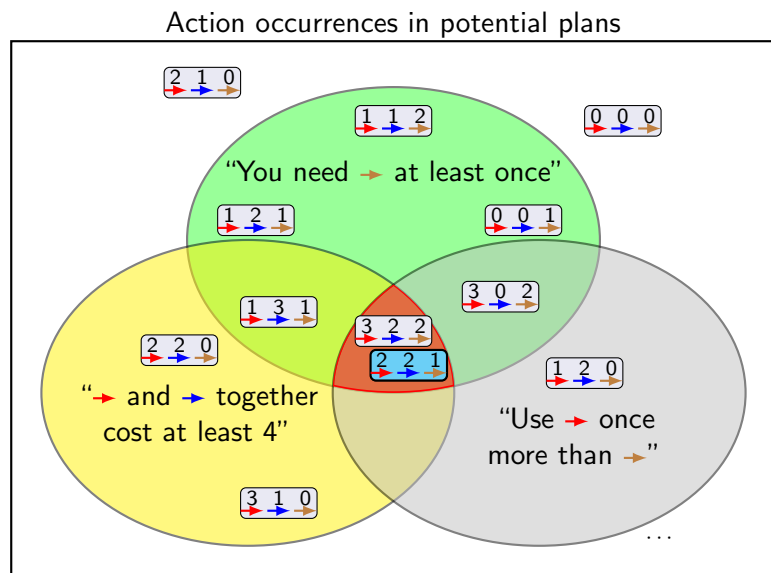
Examples:

- $\text{Count}_{a_1} + \text{Count}_{a_2} \geq 1$ “must use a_1 or a_2 at least once”
- $\text{Count}_{a_1} - \text{Count}_{a_3} \leq 0$ “cannot use a_1 more often than a_3 ”

Motivation:

- declarative way to represent knowledge about solutions
- allows reasoning about solutions to derive heuristic estimates

Operator Counting Heuristics



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Operator-counting Constraint

Definition (Operator-counting Constraints) Let Π be an FDR task with actions A and let s be a state. Let \mathcal{V} be the set of integer variables $Count_a$ for each $a \in A$. A linear inequality over \mathcal{V} is called an *operator-counting constraint* for s if for every plan π for s setting each $Count_a$ to the number of occurrences of a in π is a feasible variable assignment.

→ An operator-counting constraint must be true in every plan!.

What constraints we know already?

- Landmark constraint for landmark L : $\sum_{a \in L} Count_a \geq 1$
- Post-hoc Optimization constraint for heuristic h

$$\sum_{a \text{ is relevant for } h} Count_a \cdot c(a) \geq h(s)$$

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Flow Heuristic Constraints

Flow heuristic Use one flow constraint per fact (v, d)

$$\sum_{(v, d) \text{ produced by } a} Count_a - \sum_{(v, d) \text{ consumed by } a} Count_a \geq [G[v] = d] - [s[v] = d]$$

where:

- (v, d) consumed by a if $(v, d) \in pre(a)$ and $(v, d') \in eff(a)$ for some $d' \neq d$
- (v, d) produced by a if $(v, d) \in eff(a)$ and $((v, d) \notin pre(a))$
- $[G[v] = d] = 1$ if $G[v] = d$ and 0 otherwise
- $[s[v] = d] = 1$ if $s[v] = d$ and 0 otherwise

Remark: In Transition Normal Form tasks (see Conclusion section) we can make this constraints tighter by replacing \geq by $=$.

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Operator-counting IP/LP Heuristic

Definition (Operator-counting IP/LP Heuristic) The operator-counting integer program IP_C for a set C of operator-counting constraints for state s is

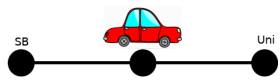
$$\text{Minimize } \sum_{a \in A} Count_a \cdot c(a) \text{ subject to}$$

$$C \text{ and } Count_a \geq 0 \text{ for all } a \in A,$$

The IP heuristic h_C^{IP} is the objective value of IP_C , the LP heuristic h_C^{LP} is the objective value of its LP-relaxation. If the IP/LP is infeasible, the heuristic estimate is ∞ .

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Questionnaire



- Variables: $at : \{SB, Home, Uni\}; v(SB) : \{T, F\}, v(Uni) : \{T, F\}$.
- Actions: $drive(x, y)$ where x, y have a road.
- Initial state: $at = Home, v(SB) = F, v(Uni) = F$.
- Goal: $at = Home, v(SB) = T, v(Uni) = T$.

Question!

What heuristic we obtain from the operator counting heuristic with the flow constraints?

- 1 For $v(Uni)$: $Count_{dr(H,Uni)} \geq 1$
- 2 For $v(SB)$: $Count_{dr(H,SB)} \geq 1$
- 3 For $at(Home)$:

$$(Count_{dr(H,SB)} + Count_{dr(H,Uni)}) - (Count_{dr(Uni,H)} + Count_{dr(SB,H)}) \geq 0$$

$\rightarrow h = 4$ (We have similar constraints for $at(SB)$ and $at(Uni)$ but they are not relevant for h in this case so we omit them)

Admissibility

Theorem (Operator-counting Heuristics are Admissible)

The IP and the LP heuristic are **admissible**.

Proof.
 Let C be a set of operator-counting constraints for state s and π be an optimal plan for s . The number of action occurrences of π are a feasible solution for C . As the IP/LP minimizes the total plan cost, the objective value cannot exceed the cost of π and is therefore an admissible estimate. □

Dominance

Theorem Let C and C' be operator-counting constraints for s and let $C \subseteq C'$. Then $IP_C \leq IP_{C'}$ and $LP_C \leq LP_{C'}$.

Proof: Every feasible solution of C' is also feasible for C . As the LP/IP is a minimization problem, the objective value subject to C can therefore not be larger than the one subject to C' .

Adding more constraints can only improve the heuristic estimate.

Theorem: Combining operator-counting heuristics in one LP is equivalent to computing their optimal general cost partitioning.

Proof idea: The linear programs are each others duals.

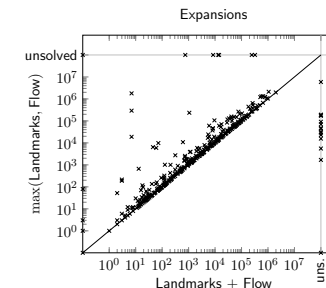
Even better combination of heuristics with IP heuristic

- Considers that actions cannot be used 1.5 times
- But computation is **no longer polynomial**

Combining Heuristics

Combination of two heuristics

- Use both operator-counting constraints
- Combination always **dominates individual heuristics**
- **Positive interaction** between constraints



Combination often better than best individual heuristic

Motivation

- Operator-counting heuristics solve an LP to compute the heuristic estimate **for a single state**.
- Can we also define an **entire heuristic function** solving only one LP?
- **Axiomatic approach** for defining heuristics:
 - What should a heuristic look like mathematically?
 - Which properties should it have?
- Define a **space of interesting heuristics**.
- Use **optimization** to pick a good representative.

Potential Heuristics

Potential Heuristics: Idea

Heuristic design as an optimization problem:

- Define simple numerical **state features** f_1, \dots, f_n .
- Consider heuristics that are **linear combinations** of features:

$$h(s) = w_1 f_1(s) + \dots + w_n f_n(s)$$

with weights (**potentials**) $w_i \in \mathbb{R}$

- Find potentials for which h is admissible and well-informed.

Motivation:

- **declarative approach** to heuristic design
- heuristic **very fast to compute** if features are

Potential Heuristics

Definition (feature)

A (state) **feature** for a planning task is a numerical function defined on the states of the task: $f : S \rightarrow \mathbb{R}$.

Definition (potential heuristic)

A **potential heuristic** for a set of features $\mathcal{F} = \{f_1, \dots, f_n\}$ is a heuristic function h defined as a **linear combination** of the features:

$$h(s) = w_1 f_1(s) + \dots + w_n f_n(s)$$

with weights (**potentials**) $w_i \in \mathbb{R}$.

↪ cf. **evaluation functions** for board games like chess

Atomic Potential Heuristics

Atomic features test if some atom is true in a state:

Definition (atomic feature)

Let $X = x$ be an atom of a FDR planning task.

The **atomic feature** $f_{X=x}$ is defined as:

$$f_{X=x}(s) = \begin{cases} 1 & \text{if variable } X \text{ has value } x \text{ in state } s \\ 0 & \text{otherwise} \end{cases}$$

- We only consider **atomic** potential heuristics, which are based on the set of all atomic features.
- **Example** for a task with state variables X and Y :

$$h(s) = 3f_{X=a} + \frac{1}{2}f_{X=b} - 2f_{X=c} + \frac{5}{2}f_{Y=d}$$

How to Set the Weights?

We want to find **good** atomic potential heuristics:

- admissible
- consistent
- well-informed

How to achieve this? **Linear programming to the rescue!**

Admissible and Consistent Potential Heuristics

Constraints on potentials **characterize** (= are necessary and sufficient for) admissible and consistent atomic potential heuristics:

Goal-awareness

$$\sum_{\text{goal facts } f} w_a = 0$$

Consistency

$$\sum_{f \text{ consumed by } a} w_a - \sum_{f \text{ produced by } a} w_a \leq c(a) \quad \text{for all actions } a$$

Remarks:

- assumes transition normal form (not a limitation)
- goal-aware and consistent = admissible and consistent

Well-Informed Potential Heuristics

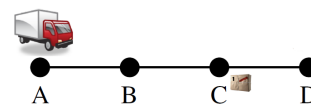
How to find a **well-informed** potential heuristic?

↪ encode **quality metric** in the **objective function** and use LP solver to find a heuristic maximizing it

Examples:

- maximize **heuristic value of a given state** (e.g., initial state)
- maximize average heuristic value of **all states** (including unreachable ones)
- maximize average heuristic value of some **sample states**
- minimize **estimated search effort**

Example: Potential Heuristic



- Initial state $I: t(A), p(C)$.
- Goal $G: t(A), p(D)$.
- Actions $A: dr(X, Y), lo(X), ul(X)$.

Goal Awareness:

$$w_{t(A)} + w_{p(D)} = 0$$

Consistency:

$$w_{at(X)} - w_{at(Y)} \leq 1 \quad (\text{drive } (X, Y))$$

$$w_{p(X)} - w_{p(T)} \leq 1 \quad (\text{load } (X))$$

$$w_{p(T)} - w_{p(X)} \leq 1 \quad (\text{unload } (X))$$

→Some Solution: $w_{p(T)} = 1, w_{p(A)} = w_{p(B)} = w_{p(C)} = 2, \text{ others} = 0$

Summary

- **Post-hoc optimization heuristic** explores the middle ground between canonical heuristic and optimal cost partitioning.
- For the same pattern collection the post-hoc optimization heuristic **dominates the canonical heuristic**.
- The computation can be done in **polynomial time**.
- Many heuristics can be formulated in terms of **operator-counting constraints**.
- The operator-counting heuristic framework allows to **combine the constraints** and to reason on the entire encoded declarative knowledge.
- The heuristic estimate for the combined constraints **can be better than the one of the best ingredient heuristic** but never worse.

Summary

- The combination into one operator-counting heuristic corresponds to the computation of the **optimal general cost partitioning for the ingredient heuristics**.
- General cost partitioning, operator-counting constraints and potential heuristics are **facets of the same phenomenon**.
- Study of each reinforces understanding of the others.

Transition Normal Form

Definition (Transition Normal Form) An FDR planning task $\Pi = (V, A, c, I, G)$ is in transition normal form (TNF) if for all $a \in A$, $V[pre(a)] = V[eff(a)]$, and $V[G] = V$.

→An operator in TNF must mention the same variables in the precondition and effect, and a goal in TNF must mention all variables (= specify exactly one goal state).

Any FDR task can be transformed into TNF:

- 1 For every variable v , add a new auxiliary value u to its domain.
- 2 For every variable v and value $d \in D_v \setminus \{u\}$, add a new operator to change the value of v from d to u with no precondition at no cost.
- 3 For all actions, if $v = d$ is a precondition with no effect on v , just add the effect $v := d$.
- 4 For all actions a and all variables $v \in V[eff(a)] \setminus V[pre(a)]$, add the precondition $v = u$ to $pre(a)$.
- 5 For every variable $v \in V \setminus V[G]$, add the condition $v = u$ to G .

Reading

- *Getting the Most Out of Pattern Databases for Classical Planning* [Pommerening *et al.* (2013)].
 Available at:
<https://ai.dmi.unibas.ch/papers/pommerening-et-al-ijcai2013.pdf>
 Content: Introduces the post-hoc optimization heuristics.
- *LP-Based Heuristics for Cost-Optimal Planning*. [Pommerening *et al.* (2014)] Available at:
<https://ai.dmi.unibas.ch/papers/pommerening-et-al-icaps-14.pdf>
 Content: Introduces the operator counting heuristics.

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Florian Pommerening, Gabriele Röger, Malte Helmert, and Blai Bonet. LP-based heuristics for cost-optimal planning. In Steve Chien, Minh Do, Alan Fern, and Wheeler Ruml, editors, *Proceedings of the 24th International Conference on Automated Planning and Scheduling (ICAPS'14)*, pages 226–234. AAAI Press, 2014.