AI Planning

11. Partial Delete Relaxation

How to (Systematically!) Take Some Delete Effects Into Account

Álvaro Torralba, Cosmina Croitoru

SAARLAND UNIVERSITY
COMPUTER SCIENCE

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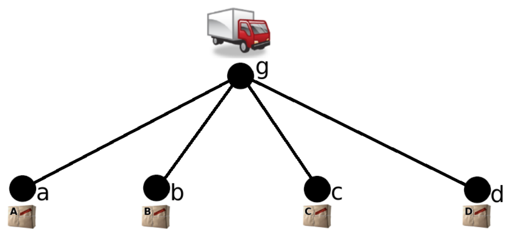
Agenda

Take This, h^+! "Star-Shape Logistics"



- State variables: $v_T : \{g, a, b, c, d\}$; $v_A, v_B, v_C, v_D : \{t, g, a, b, c, d\}$.
- Initial state: $v_T = g, v_A = a, v_B = b, v_C = c, v_D = d$.
- Goal: $v_A = g, v_B = g, v_C = g, v_D = g$.
- Actions (unit costs): $drive(x, y), load(x, y), unload(x, y)$.
  E.g., $load(x, y)$ has precondition $v_T = y, v_x = y$ and effect $v_x = t$.

→ Relaxed plan for this task: $drive(g, a)$, $drive(g, b)$, $drive(g, c)$, $drive(g, d)$, $load(A, a)$, $load(B, b)$, $load(C, c)$, $load(D, d)$, $unload(A, g)$, $unload(B, g)$, $unload(C, g)$, $unload(D, g)$. Thus: $h^+ = 12 < 16 = h^*$.

→ And with 100 star-leaf locations & packages?

Quo Vadis, h^+?

Major weaknesses of the delete relaxation:

- Completely unable to account for "to-and-fro" (cf. previous slide).
- Completely unable to account for "harmful side effects" (such as fuel consumption as a side effect of driving a truck, cf. "fill up on gas once, keep driving forever ...").

"Taking some deletes into account":

- h^+: Extreme case were no deletes are taken into account. (Fast approximations, but has the weaknesses above.)
- h^*: Extreme case were all deletes are taken into account. (Perfect, but computing it would entail solving the task in the first place.)
- Partial delete relaxation interpolates between these extremes, to obtain a fast *and* good heuristic.
  → "Interpolate" = Ability to scale smoothly from h^+ all the way to h^*.
- Challenge since 2001, first achieved in 2012 (!)

Our Agenda for This Chapter

2. **Red-Black Planning:** Introduces the most recent and, arguably, most natural idea for interpolating between h^+ and h^*: Relax only some of the FDR state variables.
3. **(A Brief Glimpse of) The Complexity of Red-Black Planning:** How many state variables do we need to relax for the heuristic computation to become tractable?
4. **Red-Black Plan Heuristics in Practice:** Naïve approaches exhibit severe over-approximation. Here's how to do better.
5. **Other Methods:** A brief glimpse at the two other known partial delete relaxation methods.

Red-Black Planning

→ Black variables switch between values ("real semantics)", red variables accumulate them ("relaxed semantics").

Definition (Red-Black Planning). *A red-black planning task is a tuple $\Pi^{\mathsf{RB}} = (V^{\mathsf{B}}, V^{\mathsf{R}}, A, c, I, G)$ where V^{B} is a set of black variables, V^{R} is a set of red variables, and everything else is exactly as for FDR tasks. The semantics is:*

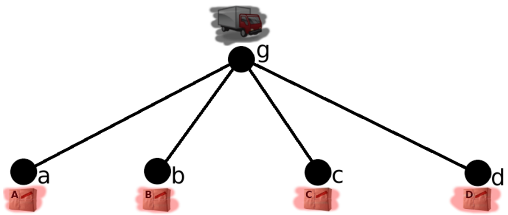
- *A state s assigns each $v \in V^{\mathsf{B}} \cup V^{\mathsf{R}}$ a subset $s(v) \subseteq D_v$, where $|s(v)| = 1$ for all $v \in V^{\mathsf{B}}$.*
- *Action a is applicable in s if $pre_a(v) \in s(v)$ for all v s.t. $pre_a(v)$ is defined.*
- *Applying a in s changes the value of black effect variables v to $\{eff_a(v)\}$, and changes the value of red effect variables v to $s(v) \cup \{eff_a(v)\}$.*
- *A state s is a goal state if $G[v] \in s(v)$ for all v s.t. $G(v)$ is defined.*

Given an FDR task $\Pi = (V, A, c, I, G)$ and a subset $V^{\mathsf{R}} \subseteq V$ of variables, the red-black relaxation of Π is the red-black task $\Pi^{\mathsf{RB}} = (V \setminus V^{\mathsf{R}}, V^{\mathsf{R}}, A, c, I, G)$. A plan for Π^{RB} is a red-black plan for Π.

Notation: $h^{*\mathsf{RB}} : S \mapsto \mathbb{R}_0^+ \cup \{\infty\}$ is the cost of an optimal red-black plan for s.

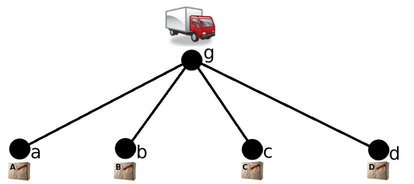
Red-Black Planning in Star-Shape Logistics

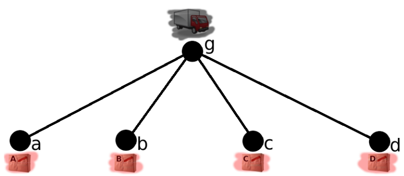
Idea: The truck moves to-and-fro, so h^+ loses information with respect to variable v_T. Let's see what happens when we paint v_T black.



- Black State variables: $v_T : \{g, a, b, c, d\}$.
- Red State variables: $v_A, v_B, v_C, v_D : \{t, g, a, b, c, d\}$.
- Initial state: $v_T = g, v_A = a, v_B = b, v_C = c, v_D = d$.
- Goal: $v_A = g, v_B = g, v_C = g, v_D = g$.
- Actions (unit costs): $drive(x, y), load(x, y), unload(x, y)$.
  E.g., $load(x, y)$ has precondition $v_T = y, v_x = y$ and effect $v_x = t$.

Red-Black Planning in Star-Shape Logistics, ctd.





Relaxed plan:

1. Initial state: $\{v_T = g, \dots\}$.
2. Apply $drive(g, a)$: $\{v_T = g, v_T = a, \dots\}$.
3. Apply $drive(g, b)$: $\{v_T = g, v_T = a, v_T = b, \dots\}$.
4. ...

Red-black plan:

1. Initial state: $\{v_T = g, \dots\}$.
2. Apply $drive(g, a)$: $\{v_T = a, \dots\}$.
3. Apply $drive(g, b)$:

Basic Observations About Red-Black Planning

Reminder: Given an FDR task (V, A, c, I, G) and a subset $V^{\mathsf{R}} \subseteq V$ of variables, the red-black relaxation of Π is $(V \setminus V^{\mathsf{R}}, V^{\mathsf{R}}, A, c, I, G)$.

- If we set $V^{\mathsf{R}} := V$, then $h^{*\mathsf{RB}} =$
- If we set $V^{\mathsf{R}} := \emptyset$, then $h^{*\mathsf{RB}} =$

→ Red-black planning allows to naturally interpolate between h^+ and h^*.

→ So, that's it? In our planner, we'll set $V^{\mathsf{R}} := \emptyset$ and be done?

Questionnaire

Question!

What if, in Star-Shape Logistics, instead of the truck we paint the packages black?

(A): $h^{*\mathsf{RB}} = h^*$

(B): $h^{*\mathsf{RB}} = h^+$

(C): We can't paint the packages black

(D): Honestly, I don't care what color the packages have

"How Many Variables do We Have to Paint Red" = All??

Theorem (Hardness for a Single Black Variable). *The problem of deciding, given a red-black planning task $\Pi^{\mathsf{RB}} = (V^{\mathsf{B}}, V^{\mathsf{R}}, A, c, I, G)$ where $|V^{\mathsf{B}}| = 1$, whether Π^{RB} is solvable, is* **NP***-complete.*

Proof Sketch. (Membership: Omitted) Hardness: By reduction from SAT.

- Red variables: For each variable $v_i \in \{v_1, \dots, v_m\}$ in the CNF, a variable v_i with domain $D_{v_1} = \{none, true, false\}$: Has v_i been assigned yet? And to which value? Initially $v_i = none$.
  For each clause $c_j \in \{c_1, \dots, c_n\}$ in the CNF, a Boolean variable sat_j: Has clause j been satisfied yet? Initially, sat_j is false; the goal requires it to be true.
- Black variable: v_0 with domain $D_{v_0} = \{1, \dots, n+1\}$: Whose variable's turn is it to be assigned? Initially, $v_0 = 1$.
- Actions that allow setting v_i from $none$ to either $true$ or $false$, provided that $v_0 = i$; apart from setting v_i, the actions also set $v_0 := i + 1$.
- Actions that allow to make sat_j true provided one of its literals has already been assigned to the correct truth value.

Simple Structure, Part I: The Black Causal Graph

The theorem holds for worst-case structure of the black variables.

→ To the rescue: Choose the red variables so that the structure of the black variables is "simple"!

Definition (Black Causal Graph). *Let $\Pi^{\mathsf{RB}} = (V^{\mathsf{B}}, V^{\mathsf{R}}, A, c, I, G)$ be a red-black planning task. The black causal graph of Π^{RB} is the directed graph with vertices V^{B} and an arc (u, v) whenever there exists an action $a \in A$ so that either (i) there exists $a \in A$ so that $pre_a(u)$ and $eff_a(v)$ are both defined, or (ii) there exists $a \in A$ so that $eff_a(u)$ and $eff_a(v)$ are both defined.*

→ The subgraph of the causal graph induced by the black variables.

→ The black causal graph in Star-Shape Logistics:

Simple Structure, Part II: Invertible Variables

Reminder: → Chapter 5

Let $\Pi = (V, A, c, I, G)$ be an FDR planning task, and let $v \in V$. The domain transition graph (DTG) of v is the arc-labeled directed graph with vertices D_v, and, for every $d, d' \in D_v$ and $a \in A$ where either (i) $pre_a(v) = d$ and $eff_a(v) = d'$ or (ii) $pre_a(v)$ is not defined and $eff_a(v) = d'$, an arc $d \xrightarrow{a} d'$.
We refer to $d \xrightarrow{a} d'$ as a value transition of v. We write $d \xrightarrow{a}_\varphi d'$ where $\varphi = pre_a \setminus \{v = d\}$ is the outside condition.

Let $d \rightarrow_\varphi d'$ be a value transition of v. We say that $d \rightarrow_\varphi d'$ is invertible if there exists a value transition $d' \rightarrow_{\varphi'} d$ where $\varphi' \subseteq \varphi$.

Notation: A variable is invertible if all transitions in its DTG are invertible.

→ The DTG of the truck variable v_T in Star-Shape Logistics:

The SMS Theorem

Theorem ("The SMS Theorem"). *Let $\Pi = (V, A, c, I, G)$ be an FDR planning task, and let $V^{\mathsf{R}} \subseteq V$ be a subset of its state variables. Say that, in the red-black relaxation of Π, the black causal graph does not contain any arcs, and all black variables are invertible. Then any relaxed plan for Π can in polynomial time be transformed into a red-black plan for Π.*

- **Idea:** Relaxed Plan Repair. Execute the relaxed plan step-by-step. If a black precondition (or goal) is not satisfied, we can move each black variable concerned into its required precondition/goal value separately.
- **Corollary (a):** If a relaxed plan exists, we can easily generate a red-black plan. **Trivial (b):** If no relaxed plan exists, then no red-black plan can exist either. From (a) + (b), we have a complete and efficient red-black planning procedure.
- **Usage:** On any state s encountered during search, generate a red-black plan for s and take its cost as the heuristic value. (= "In h^{FF}, replace relaxed plan by red-black plan.")

Relaxed Plan Repair: Idea

By the SMS Theorem's prerequisites:

(a) Every black variable is invertible. E.g., truck can always move back directly.
(b) Every action moves at most one black variable.
(c) If a moves a black variable v, all outside conditions on $v' \neq v$ are red.
E.g., $drive(x, y)$ has precondition $v_T = x$ and effect $v_T = y$.
E.g., if we paint the truck red and the packages black, $load(x, y)$ has precondition $v_T = y, v_x = y$ and effect $v_x = t$.

Relaxed plan repair algorithm: Assume relaxed plan $\vec{a}^+ = \langle a_1, \dots, a_n \rangle$

- $s :=$ red-black outcome of a_1 in initial state.
- For any black v, if $s(v) \neq z$ precondition of a_2: Move v to value z.
  → (a) Path exists, as v is invertible: Go back to $I(v)$, then follow $\vec{a}^+$ to z.
  → (b) Moving v does not affect any other black variables.
  → (c) All outside conditions used by the path are red; and have already been achieved during our execution so far, thus they are true.
- $s :=$ red-black outcome of a_2. Proceed with $\langle a_3, \dots, a_n \rangle$ and the goal.

Relaxed Plan Repair: Pseudo-Code

// $\Pi = (V, A, c, I, G)$, relaxed plan $\vec{a}^+ = \langle a_1, \dots, a_n \rangle$, black and red variables V^{B}, V^{R}
$\vec{a} := \langle a_1 \rangle$; $s := I[\![a_1]\!]$ *// red-black semantics (slide 8)*
for $i = 2$ to n **do** *// Repair black action preconditions*
  if $pre_{a_i}(V^{\mathsf{B}}) \not\subseteq s$ **then**
    $\vec{a}^{\mathsf{B}} :=$ Achieve$(s, pre_{a_i}(V^{\mathsf{B}}))$; $\vec{a} := \vec{a} \circ \vec{a}^{\mathsf{B}}$; $s := s[\![\vec{a}^{\mathsf{B}}]\!]$
  endif
  $\vec{a} := \vec{a} \circ \langle a_i \rangle$; $s := s[\![a_i]\!]$
endfor
if $G(V^{\mathsf{B}}) \not\subseteq s$ **then** *// Repair black goals*
  $\vec{a}^{\mathsf{B}} :=$ Achieve$(s, G(V^{\mathsf{B}}))$; $\vec{a} := \vec{a} \circ \vec{a}^{\mathsf{B}}$
endif
return $\vec{a}$

Procedure: Achieve(s, g)
$\vec{a}^{\mathsf{B}} := \langle\rangle$
for $v \in V^{\mathsf{B}}$ s.t. $g(v)$ is defined **do** *// Move black variables into place separately*
  $\vec{a}^{\mathsf{B}} := \vec{a}^{\mathsf{B}} \circ$ invert path used by $\vec{a}$ from $I(v)$ to $s(v)$
  $\vec{a}^{\mathsf{B}} := \vec{a}^{\mathsf{B}} \circ$ path used by $\vec{a}^+$ from $I(v)$ to $g(v)$
endfor
return $\vec{a}^{\mathsf{B}}$

Questionnaire

Relaxed Plan Repair in Star-Shape Logistics

Questionnaire

Question!

Does Relaxed Plan Repair yield an accurate heuristic function?

(A): Yes (B): No

How to Choose the Red Variables?

Input: A planning task $\Pi = (V, A, I, G)$

Output: Partitioning of V into V^{B} and V^{R}

Method:

1. Compute the black causal graph, and the DTG for each $v \in V$
2. Initialize $V^{\mathsf{B}} := V$ and $V^{\mathsf{R}} := \emptyset$
3. For all $v \in V^{\mathsf{B}}$: if v is not invertible then $V^{\mathsf{B}} := V^{\mathsf{B}} \setminus \{v\}$, $V^{\mathsf{R}} := V^{\mathsf{R}} \cup \{v\}$
4. While black causal graph contains arc (v, v') between $v, v' \in V^{\mathsf{B}}$ do: (*) choose $w \in \{v, v'\}$; $V^{\mathsf{B}} := V^{\mathsf{B}} \setminus \{w\}$, $V^{\mathsf{R}} := V^{\mathsf{R}} \cup \{w\}$

→ How to make the choice (*)?

Questionnaire

Consider the same relaxed plan: $drive(g, a)$, $drive(g, b)$, $drive(g, c)$, $drive(g, d)$, $load(A, a)$, $load(B, b)$, $load(C, c)$, $load(D, d)$, $unload(A, g)$, $unload(B, g)$, $unload(C, g)$, $unload(D, g)$.

Question!

What does Relaxed Plan Repair do if, in Star-Shape Logistics, instead of the truck we paint the packages black?

(A): Nothing (B): Same as Before

The Problem, and a Solution

What is the problem?

- **Relaxed Plan:** $drive(g, a), drive(g, b), drive(g, c), drive(g, d)$, $load(A, a), load(B, b), load(C, c), load(D, d)$, $unload(A, g), unload(B, g), unload(C, g), unload(D, g)$.
- The relaxed plan can (and will) schedule all truck moves first. We can't.
- In general: Commitments made by relaxed plan throw us off in red-black.

What can we do about it? Let's rely *less* on the relaxed plan!

- $R^+ := [G(V^R) \cup \bigcup_{a \in \vec{a}^+} pre_a(V^R)] \setminus I$ where $\vec{a}^+$ is a relaxed plan: The red precondition/goal values achieved along the relaxed plan.
- In the example: $R^+ = \{v_A = t, v_A = g, v_B = t, v_B = g, v_C = t, v_C = g, v_D = t, v_D = g\}$
- **Idea:** Keep selecting actions that achieve one more fact from R^+!

  $\rightarrow$ In the example, these actions will be the loads/unloads, and the truck moves will simply be inserted as a helper for achieving their preconditions.

Relaxed Facts Following: Outline

Notation:

- R: Red values already true, i.e., true in the outcome state s of the current red-black plan prefix (under red-black execution semantics).
- B: Overall set of black values $v = d$ reachable from $I(v)$ using only outside conditions from R.

Algorithm sketch:

- $s := I$. If $R \supseteq R^+$ then stop.
- Select a from $A' := \{a \mid pre_a \subseteq R \cup B, eff_a \cap (R^+ \setminus R) \neq \emptyset\}$.
- For any black v, if $s(v) \neq z$ precondition of a: Move v to value z.
  $\rightarrow$ Path exists, can be executed in s, and does not affect any other black variables: Similar arguments as for Relaxed Plan Repair.
- $s :=$ red-black outcome of a. Proceed with the rest of R^+.
- Move all black goal variables into place.
  $\rightarrow$ Possible because all of R^+, and thus all necessary outside conditions for these paths, have been achieved.

Relaxed Facts Following in Star-Shape Logistics

Relaxed Facts Following: Pseudo-Code

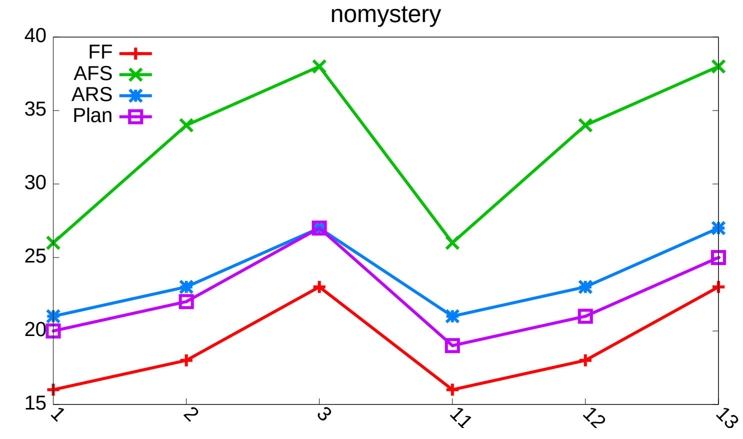
$\vec{a} := \langle\rangle$; $s := I$; UpdateRB()
while $R \not\supseteq R^+$ **do** *// Achieve one more R^+ fact*
  $A' := \{a \mid pre_a \subseteq R \cup B, eff_a \cap (R^+ \setminus R) \neq \emptyset\}$
  Select $a \in A'$
  if $pre_a(V^B) \not\subseteq s$ **then**
    $\vec{a}^B :=$ Achieve$(s, pre_a(V^B))$; $\vec{a} := \vec{a} \circ \vec{a}^B$; $s := s[\![\vec{a}^B]\!]$ *// red-black semantics*
  endif
  $\vec{a} := \vec{a} \circ \langle a \rangle$; $s := s[\![a]\!]$; UpdateRB()
endwhile
if $G(V^B) \not\subseteq s$ **then** *// Repair black goals*
  $\vec{a}^B :=$ Achieve$(s, G(V^B))$; $\vec{a} := \vec{a} \circ \vec{a}^B$
endif
return $\vec{a}$

Procedure: UpdateRB() *// Update content of R and B*
$R := s(V^R)$; $B := \emptyset$
for $v \in V^B$ **do**
  $B := B \cup$values reachable in v's DTG from $I(v)$ using only outside conditions from R
endfor

Procedure: Achieve(s, g) *// Same as slide 19*

Reduced Over-Estimation

Initial state heuristic values:



[FF: h^{FF}; AFS: Relaxed Plan Repair; ARS: Relaxed Facts Following]

Improved Performance

Coverage (instances solved), for painting strategies "A" vs. "C":

	#	FF	AF	AR	CF	CR
barman	20/20	15	16	16	17	2
depot	22/22	15	14	15	14	15
driverlog	20/20	18	16	18	17	18
elevators	20/20	17	14	13	2	11
floortile	20/20	4	6	3	6	3
grid	5/5	4	3	4	4	4
logistics98	35/35	22	5	35	5	35
mprime	35/35	30	31	30	29	30
nomystery	20/20	8	7	14	7	14
parcprinter	13/20	4	6	4	6	4
Pipes-notank	40/50	20	18	18	18	18
Pipes-tank	40/50	14	16	12	16	13
rovers	40/40	23	16	25	17	25
satellite	36/36	23	22	28	22	28
scanalyzer	14/20	10	12	14	10	10
sokoban	20/20	19	19	19	18	19
tidybot	20/20	15	14	13	16	13
tpp	30/30	20	15	20	15	20
transport	20/20	0	0	0	1	0
trucks	30/30	16	15	16	16	14
visitall	20/20	5	3	17	3	17
woodworking	20/20	2	2	3	2	3
$\sum$	891/926	644	610	677	601	656

[AF, CF: Relaxed Plan Repair; AR, CR: Relaxed Facts Following]

Questionnaire

Consider the same relaxed plan: $drive(g, a)$, $drive(g, b)$, $drive(g, c)$, $drive(g, d)$, $load(A, a)$, $load(B, b)$, $load(C, c)$, $load(D, d)$, $unload(A, g)$, $unload(B, g)$, $unload(C, g)$, $unload(D, g)$.

Question!

What does Relaxed Facts Following do if, in Star-Shape Logistics, instead of the truck we paint the packages black?

(A): Nothing (B): Same as Before

Before We Begin

Which Other Methods? Apart from red-black relaxation, there are two other methods that allow to smoothly interpolate between h^+ and h^*:

- **Variable Pre-Merging:** Use $h^+_{\Pi^M}$ where Π^M is obtained from Π by merging a subset M of variables into a single variable.
- **Conjuncts Compilation:** Use $h^+_{\Pi^C}$ where Π^C is obtained from Π by explicitly representing a subset C of fact conjunctions.

Illustrative example we will use here: Buy-A-Car

VS.

- State variables: C, G : Boolean.
- Initial state: $C = 0, G = 1$.
- Goal: $C = 1, G = 1$.
- Action: $buy()$
  Precondition $C = 0, G = 1$; effect $C = 1, G = 0$.

So what? Task is unsolvable but has relaxed plan $buy()$.

Variable Pre-Merging

Method outline:

- Before planning starts, select a subset $M \subseteq V$ of FDR variables.
- Compute the DTG of a merged variable x_M equivalent to the cross-product of M.
- Replace M with x_M in the planning task Π to obtain the merged task Π^M.

Applied to Buy-A-Car:

- $M := \{C, G\}$; $D_{x_M} = \{C0G0, C0G1, C1G0, C1G1\}$.
- $I(x_M) =$; $G(x_M) =$.
- Value transitions on x_M:
- Relaxed plan for Π^M:

Variable Pre-Merging: Convergence

Proposition (Variable Pre-Merging is Perfect in the Limit). *Let $\Pi = (V, A, c, I, G)$ be an FDR planning task. If we set $M := V$ in the above, then $h^+_{\Pi^M} = h^*_\Pi$.*

Proof. If we merge all variables, then the merged task has a single variable whose DTG is the overall state space. A relaxed plan through that DTG is a solution path in the state space, QED.

→ Problem with that result?

Explicit Conjunctions: Idea

Method outline:

- Before planning on FDR task $\Pi = (V, A, c, I, G)$ starts, select a subset C of fact conjunctions c to be represented explicitly using new π-fluents π_c.
  → E.g., $C = \{p \wedge q, g_1 \wedge g_2\}$ and we introduce new Boolean variables $\pi_{p \wedge q}$ and $\pi_{g_1 \wedge g_2}$.
- Construct a compiled task Π^C, modifying Π to correctly account for the intended semantics of each π_c.
- Initial state: Include those π_c where $c \subseteq I$. (We identify conjunctions with sets of facts.)
- Action effects: If eff_a intersects c and does not contradict c, then make a copy of a whose effect includes π_c.
- Action preconditions and goal: In Π^C, include each π_c into every condition (precondition/goal) that contains c.

Explicit Conjunctions in Buy-A-Car

vs.

- State variables: v_C, v_G : Boolean.
- Initial state: $v_C = 0, v_G = 1$.
- Goal: $v_C = 1, v_G = 1$.
- Actions (unit costs): $buy()$
  Precondition $v_C = 0, v_G = 1$; effect $v_C = 1, v_G = 0$.

Now let's make one conjunction explicit:

- C := set of conjunctions containing only $c := v_C = 1 \wedge v_G = 1$.
- Goal of Π^C:
- Actions a where eff_a intersects c and does not contradict c:
- Relaxed plan for Π^C:

The Π^C Compilation

Shorthand notation: For fact set X, $X^C := X \cup \{\pi_c \mid c \in C, c \subseteq X\}$.

Definition (The Π^C compilation). *Let $\Pi = (V, A, c, I, G)$ be an FDR planning task, and let C be a set of conjunctions (fact sets/partial variable assignments) in Π. Then Π^C is the task $(V^C, A^C, c^C, I^C, G^C)$ where:*

- *$V^C = V \cup \{\pi_c \mid c \in C\}$, each π_c being a new Boolean variable.*
- *A^C contains an action $a^{C'}$ for every pair $a \in A$, $C' \subseteq C$ s.t., for all $c' \in C'$, $\mathit{eff}_a \cap c' \neq \emptyset$ and there is no $v \in V$ s.t. $\mathit{eff}_a(v)$ and $c(v)$ are both defined and $\mathit{eff}_a(v) \neq c(v)$; $a^{C'}$ is then given by*
  - *$\mathit{pre}_{a^{C'}} = [\mathit{pre}_a \cup \bigcup_{c' \in C'}(c' \setminus \mathit{eff}_a)]^C$, and*
  - *$\mathit{eff}_{a^{C'}} = \mathit{eff}_a \cup \{\pi_{c'} \mid c' \in C'\}$.*
- *c^C extends c to A^C by $c(a^{C'}) = c(a)$.*
- *I^C and G^C are as defined by the shorthand notation.*

→ Action a can achieve conjunctions C', at the cost of having the "missing context" $\bigcup_{c' \in C'}(c' \setminus \mathit{eff}_a)$ beforehand.

The Π^C Compilation: Why "every pair $a \in A$, $C' \subseteq C$"?

The Π^C Compilation: Example for $h^+ < h^+_{\Pi^C} < \infty$

Example from previous slide, modified to have conflict between q_1 and q_2:

Facts: $\{q_1, q_2, p, g_1, g_2\}$; initial state: $\emptyset$; goal: $\{g_1, g_2\}$. Actions:

- $a_{q_1} : \emptyset \rightarrow q_1, \neg p, \neg q_2$ $\quad a_{q_2} : \emptyset \rightarrow q_2, \neg p, \neg q_1$
- $a_p : \emptyset \rightarrow p$
- $a_{g_1} : p, q_1 \rightarrow g_1$ $\quad a_{g_2} : p, q_2 \rightarrow g_2$

→ Plan?

→ Relaxed plan?

→ Relaxed plan for Π^C when taking $C := \{p \wedge q_1, p \wedge q_2, q_1 \wedge q_2\}$?

- Can we do $a_{q_1}, a_{q_2}, a_p^{\{p \wedge q_1, p \wedge q_2\}}, a_{g_1}, a_{g_2}$?
- So how to do it?

Explicit Conjunctions: Convergence

Theorem (The Π^C Compilation is Perfect in the Limit). *Let* Π *be an FDR planning task. Then there exists* C *such that* $h^+_{\Pi^C} = h^*_\Pi$.

Proof. For sufficiently large m, $h^m_\Pi = h^*_\Pi$ **(Chapter 8)**. If we choose C to be all size-$\leq m$ conjunctions, then $h^m_\Pi = h^1_{\Pi^C}$ [see e.g. Keyder *et al.* (2012)]. Done with $h^1_{\Pi^C} = h^{\max}_{\Pi^C} \leq h^+_{\Pi^C}$ **(Chapter 9)** $\leq h^*_\Pi$.

Problem with that result: The "Limit" case, as proved here, is $h^m = h^*$ which typically happens only for prohibitively large m.

$\rightarrow$ However, the proof argument ignores the advantages of $h^+(\Pi^C)$:
1. We can choose C more freely.
2. we use h^+ instead of h^1.
So there is hope to obtain h^* with much smaller C. (See slide 47)

So Which Method Should We Use?

Summary

- The delete relaxation is unable to account for to-and-fro, and for harmful side effects. To counter-act this, we should "take some deletes into account". If such a method is able to render h^+ perfect in the limit, then we call it an interpolation method.
- Red-black planning is an interpolation method that relaxes only a subset of the FDR state variables (the red variables), keeping the others (the black variables) intact.
- Red-black planning is **NP**-hard even with a single black variable, but is tractable if we demand ("SMS Theorem") that the black causal graph is acyclic, and that all black variables are invertible.
- Naïve red-black planning by Relaxed Plan Repair is prone to over-estimation, but we can fix this by relying less on the relaxed plan in Relaxed Facts Following.
- Explicit conjunctions is an alternative interpolation method, expliciting the semantics of a subset C of conjunctions over the task's facts.

Remarks

Beyond the SMS theorem: I've treated you to this simple setup for simplicity.

- Our actual theorem is more general in requiring only an acyclic black causal graph, instead of requiring there to be no arcs at all.
- Our actual theorem is more general in requiring only "relaxed side-effects invertibility", a weaker notion of invertibility.
- There's an alternative tractability theorem, requiring only that the domain size of the (single) black variable is bounded.

Painting strategies: Which variables to paint red respectively black?

- We experimented with lots of methods based on different notions of which variables are "most important" (to be painted black as much as possible).
- The performance differences are, generally speaking, marginal.
- In fact, there typically is very little choice if we insist on painting black "as much as possible".
- Comprehensive results: [Domshlak *et al.* (2015)]

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... (a few examples) ...

Theory Understanding:

- Identify special cases where polynomial-size C can/cannot render $h^+_{\Pi^C}$ perfect.
- Deeper complexity analysis of red-black planning.
- Generalizations of red-black planning where variables may remember *some* of their values.
- Etc. ...

Alternative Uses of Partial Delete Relaxation:

- Learning to detect dead-ends [Steinmetz and Hoffmann (2016)]/learning to refine heuristic values during search.
- Incremental red-black.
- Plan templates to seed plan-space search.
- Plan-template distance heuristics.

Reading

- *Who Said we Need to Relax All Variables?* [Katz *et al.* (2013b)].

  Available at:
  http://fai.cs.uni-saarland.de/hoffmann/papers/icaps13.pdf

  Content: Introduces red-black planning and our main complexity results, along with a brief analysis of when/where $h^{*\text{RB}}$ is perfect.

- *Red-Black Relaxed Plan Heuristics* [Katz *et al.* (2013a)].

  Available at:
  http://fai.cs.uni-saarland.de/hoffmann/papers/aaai13.pdf

  Content: Simpler tractable fragment (SMS Theorem + relaxed side-effects invertibility) used to generate red-black plan heuristics.

Reading

- *Red-Black Relaxed Plan Heuristics Reloaded* [Katz and Hoffmann (2013)].

  Available at:
  http://fai.cs.uni-saarland.de/hoffmann/papers/socs13.pdf

  Content: As above, but with Relaxed Facts Following for reduced over-estimation and (much) better performance.

- *Red-Black Planning: A New Systematic Approach to Partial Delete Relaxation* [Domshlak *et al.* (2015)].

  Available at:
  http://fai.cs.uni-saarland.de/hoffmann/papers/ai15.pdf

  Content: The whole storyline of the previous three papers, comprehensively told and underfed with systematic experiments.

Reading, ctd.

- *Improving Delete Relaxation Heuristics Through Explicitly Represented Conjunctions* [Keyder *et al.* (2014)].

  Available at:
  http://fai.cs.uni-saarland.de/hoffmann/papers/jair14.pdf

  Content: Uses the Π^C compilation as well as another compilation Π^C_{ce} which employs conditional effects to avoid the exponential blow-up in $|C|$. This comes at the prize of a loss in informedness, however Π^C_{ce} is still perfect in the limit.

- *Combining the Delete Relaxation with Critical-Path Heuristics: A Direct Characterization* [Fickert *et al.* (2016)].

  Available at:
  http://fai.cs.uni-saarland.de/hoffmann/papers/jair16.pdf

  Content: Avoids the compilation altogether. Achieves the same complexity reduction as Π^C_{ce}, but without the information loss.

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