AI Planning

7. Heuristic Search

How to Avoid Having to Look at a Gazillion States

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Agenda

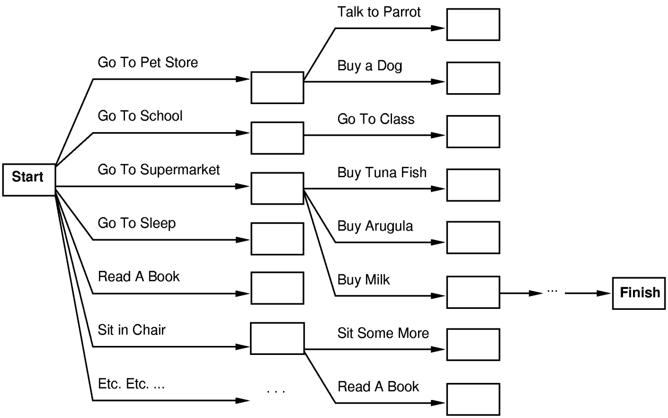
1. Introduction
2. What Are Heuristic Functions?
3. How to Use Heuristic Functions?
4. How to Obtain Heuristic Functions?
5. Conclusion

Reminder: Our Long-Term Agenda

Fill in (some) details on these choices:

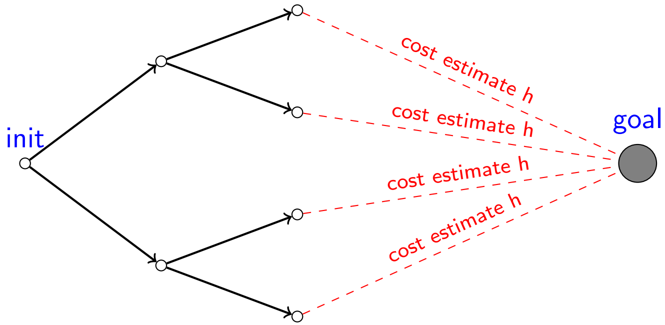
1. Search space: Progression vs. regression.
   → **Previous Chapter**
2. Search algorithm: Uninformed vs. heuristic; systematic vs. local.
   → **This Chapter**
3. Search control: Heuristic functions and pruning methods.
   → **Chapters 8–20**

Looking at a Gazillion States?



→ Use heuristic function to guide the search towards the goal!

Heuristic Search



→ Heuristic function h estimates the cost of an optimal path from a state s to the goal; search prefers to expand states s with small $h(s)$.

Live Demo vs. Breadth-First Search:

http://qiao.github.io/PathFinding.js/visual/

Our Agenda for This Chapter

2. **What Are Heuristic Functions?** Gives the basic definition, and introduces a number of important properties that we will be considering throughout the course.
3. **How to Use Heuristic Functions?** Recaps the basic heuristic search algorithms from AI'17, and adds a few new ones. Gives a few planning-specific algorithms and explanations.
4. **How to Obtain Heuristic Functions?** Recaps the concept of "Relaxation" from AI'17: A basic explanation how heuristic functions are derived in practice.

Heuristic Functions

Definition (Heuristic Function). *Let Π be a planning task with state space $\Theta_\Pi = (S, L, c, T, I, S^G)$. A heuristic function, short heuristic, for Π is a function $h : S \mapsto \mathbb{R}_0^+ \cup \{\infty\}$. Its value $h(s)$ for a state s is referred to as the state's heuristic value, or h value.*

Definition (Remaining Cost, h^*). *Let Π be a planning task with state space $\Theta_\Pi = (S, L, c, T, I, S^G)$. For a state $s \in S$, the state's remaining cost is the cost of an optimal plan for s, or ∞ if there exists no plan for s. The perfect heuristic for Π, written h^*, assigns every $s \in S$ its remaining cost as the heuristic value.*

→ Heuristic functions h estimate remaining cost h^*.

→ These definitions apply to both, STRIPS and FDR.

Heuristic Functions: The Eternal Trade-Off

What does it mean, "estimate remaining cost"?

- In principle, the "estimate" is an arbitrary function. In practice, we want it to be accurate (aka: informative), i.e., close to the actual remaining cost.
- We also want it to be fast, i.e., a small overhead for computing h.
- These two wishes are in contradiction! Extreme cases?

→ We need to trade off the accuracy of h against the overhead of computing it. → **Chapters 8–17**

→ What exactly is "accuracy"? How does it affect search performance? Interesting and challenging subject! We'll consider this in **Chapter 17**.

Questionnaire

Question!

For root-finding on a map, the straight-line distance heuristic certainly has small overhead. But is it accurate?

(A): No
(B): Yes
(C): Sometimes
(D): Maybe

Properties of Individual Heuristic Functions

Definition (Safe/Goal-Aware/Admissible/Consistent). *Let Π be a planning task with state space $\Theta_\Pi = (S, L, c, T, I, S^G)$, and let h be a heuristic for Π. The heuristic is called:*

- *safe if, for all $s \in S$, $h(s) = \infty$ implies $h^*(s) = \infty$;*
- *goal-aware if $h(s) = 0$ for all goal states $s \in S^G$;*
- *admissible if $h(s) \leq h^*(s)$ for all $s \in S$;*
- *consistent if $h(s) \leq h(s') + c(a)$ for all transitions $s \xrightarrow{a} s'$.*

$\rightarrow$ Relationships:

Proposition. *Let Π be a planning task, and let h be a heuristic for Π. If h is admissible, then h is goal-aware. If h is admissible, then h is safe. If h is consistent and goal-aware, then h is admissible. No other implications of this form hold.*

Consistency: Illustration

Properties of Individual Heuristic Functions, ctd.

Examples:

- Is h =Manhattan distance in the 15-Puzzle safe/goal-aware/admissible/ consistent? All yes. Easy for goal-aware and safe (h is never ∞). Consistency: Moving a tile can't decrease h by more than 1.
- Is h =straight line distance safe/goal-aware/admissible/consistent?
- An admissible but inconsistent heuristic: To-Moscow with $h(SB) = 1000$, $h(KL) = 100$.

→ In practice, most heuristics are safe and goal-aware, and admissible heuristics are typically consistent.

What about inadmissible heuristics?

- Inadmissible heuristics typically arise as approximations of admissible heuristics that are too costly to compute. (Examples: **Chapter 9**)

Domination Between Heuristic Functions

Definition (Domination). *Let Π be a planning task, and let h and h' be admissible heuristics for Π. We say that h' dominates h if $h \leq h'$, i.e., for all states s in Π we have $h(s) \leq h'(s)$.*

$\rightarrow$ h' dominates h = "h' provides a lower bound at least as good as h".

Remarks:

- Example: h' =Manhattan Distance vs. h =Misplaced Tiles in 15-Puzzle: Each misplaced tile accounts for at least 1 (typically, more) in h'.
- h^* dominates every other admissible heuristic.
- Modulo tie-breaking, the search space of A^* under h' can only be smaller than that under h. (See [Holte (2010)] for details)
- In **Chapter 17**, we will consider much more powerful concepts, comparing entire *families* of heuristic functions.

Additivity of Heuristic Functions

Definition (Additivity). *Let Π be a planning task, and let $h_1, \dots, h_n$ be admissible heuristics for Π. We say that $h_1, \dots, h_n$ are additive if $h_1 + \dots + h_n$ is admissible, i.e., for all states s in Π we have $h_1(s) + \dots + h_n(s) \leq h^*(s)$.*

→ An ensemble of heuristics is additive if its sum is admissible.

Remarks:

- Example: h_1 considers only tiles 1 . . . 7, and h_2 considers only tiles 8 . . . 15, in the 15-Puzzle: The two estimates are then, intuitively, "independent".
  (h_1 and h_2 are orthogonal projections → **Chapter 12**)
- We can always combine $h_1, \dots, h_n$ admissibly by taking the $\max$. Taking $\sum$ is *much* stronger; in particular, $\sum$
- In **Chapters 15–16**, we will devise a third, strictly more general, technique to admissibly combine heuristic functions.

What Works Where in Planning?

Blind (no h) **vs. heuristic:**

- For satisficing planning, heuristic search vastly outperforms blind algorithms pretty much everywhwere.
- For optimal planning, heuristic search also is better (but the difference is not as huge).

Systematic (maintain all options) **vs. local** (maintain only a few) **:**

- For satisficing planning, there are successful instances of each.
- For optimal planning, systematic algorithms are required.

$\rightarrow$ Here, we briefly cover the search algorithms most successful in planning. For more details (in particular, for blind search), refer to AI'18 Chapters 4 and 5.

Reminder: Greedy Best-First Search and A^*

For simplicity, duplicate elimination omitted and using AI'17 notation:

function Greedy Best-First Search $[A^*]$*(problem)* **returns** a solution, or failure
  node $\leftarrow$ a node n with n.*state*=*problem.InitialState*
  frontier $\leftarrow$ a priority queue ordered by ascending h $[g+h]$, only element n
  loop do
    if *Empty?(frontier)* **then return** failure
    $n \leftarrow$ *Pop(frontier)*
    if *problem.GoalTest*(n.*State*) **then return** *Solution*(n)
    for each *action* a **in** *problem.Actions*(n.*State*) **do**
      $n' \leftarrow$ *ChildNode(problem,*n,a*)*
      Insert(n', $h(n')$ $[g(n')+h(n')]$, *frontier*)

$\rightarrow$ Greedy best-first search explores states by increasing heuristic value h.
A^* explores states by increasing plan-cost estimate $g+h$.

Greedy Best-First Search: Remarks

Properties:

- Complete? Yes, with duplicate elimination. (If $h(s) = \infty$ states are pruned, h needs to be safe.)
- Optimal? No. (Even for perfect heuristics! E.g., say the start state has two transitions to goal states, one of which costs a million bucks while the other one is for free. Nothing keeps Greedy Best-First Search from choosing the bad one.)

Technicalities:

- Duplicate elimination: Insert child node n' only if n'.State is not already contained in *explored* $\cup$ States(*frontier*). (Cf. AI'17)

Bottom line: Fast but not optimal $\implies$ satisficing planning.

A^*: Remarks

Properties:

- Complete? Yes. (Even without duplicate detection; if $h(s) = \infty$ states are pruned, h needs to be safe.)
- Optimal? Yes, for admissible heuristics.

Technicalities:

- "Plan-cost estimate" $g(s) + h(s)$ known as f-value $f(s)$ of s.
  $\rightarrow$ If $g(s)$ is taken from a cheapest path to s, then $f(s)$ is a lower bound on the cost of a plan through s.
- Duplicate elimination: If n'.State$\notin$*explored* $\cup$ States(*frontier*), then insert n'; else, insert n' only if the new path is cheaper than the old one, and if so remove the old path. (Cf. AI'17)

Bottom line: Optimal for admissible h $\Longrightarrow$ optimal planning, **with such** h.

Weighted A^*

For simplicity, duplicate elimination omitted and using AI'17 notation:

function Weighted A^**(problem)* **returns** a solution, or failure
  node ← a node n with n*.state=problem.InitialState*
  frontier ← a priority queue ordered by ascending $g + W * h$, only element n
  loop do
    if *Empty?(frontier)* **then return** failure
    $n \leftarrow$ *Pop(frontier)*
    if *problem.GoalTest*(n*.State*) **then return** *Solution*(n)
    for each *action* a **in** *problem.Actions*(n*.State*) **do**
      $n' \leftarrow$ *ChildNode(problem,*n,a*)*
      Insert(n', [$g(n') + W * h(n')$], *frontier)*

→ Weighted A^* explores states by increasing weighted-plan-cost estimate $g + W * h$.

Weighted A^*: Remarks

The weight $W \in \mathbb{R}_0^+$ is an **algorithm parameter:**

- For $W = 0$, weighted A^* behaves like?
- For $W = 1$, weighted A^* behaves like? A^*.
- For $W = 10^{100}$, weighted A^* behaves like?

Properties:

- For $W > 1$, weighted A^* is bounded suboptimal.

  $\rightarrow$ If h is admissible, then the solutions returned are at most a factor W more costly than the optimal ones.

Bottom line: Allows to interpolate between greedy best-first search and A^*, trading off plan quality against computational effort.

Hill-Climbing

function Hill-Climbing **returns** a solution
  node ← a node n with n.*state*=*problem.InitialState*
  loop do
    if *problem.GoalTest*(n.*State*) **then return** *Solution*(n)
    N ← the set of all child nodes of n
    n ← an element of N minimizing h /* (random tie breaking) */

Remarks:

- Is this complete or optimal? No.
- Can easily get stuck in local minima where immediate improvements of $h(n)$ are not possible.
- Many variations: tie-breaking strategies, restarts, ... (cf. AI'17)

Enforced Hill-Climbing [Hoffmann and Nebel (2001)]

function Enforced Hill-Climbing **returns** a solution
  node ← a node n with n.*state*=*problem.InitialState*
  loop do
    if *problem.GoalTest*(n.*State*) **then return** *Solution*(n)
    Perform breadth-first search for a node n' s.t. $h(n') < h(n)$
    $n \leftarrow n'$

Remarks:

- Is this optimal? No.
- Is this complete? See next slide.

Questionnaire

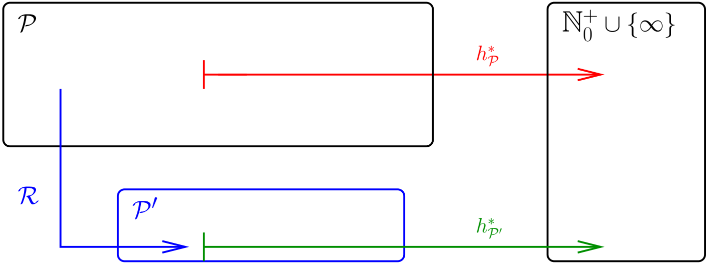
function Enforced Hill-Climbing **returns** a solution
  node $\leftarrow$ a node n with n.*state*=*problem.InitialState*
  loop do
    if *problem.GoalTest*(n.*State*) **then return** *Solution*(n)
    Perform breadth-first search for a node n' s.t. $h(n') < h(n)$
    $n \leftarrow n'$

Question!

Assume that $h(s) = 0$ if and only if s is a goal state. Is Enforced Hill-Climbing complete?

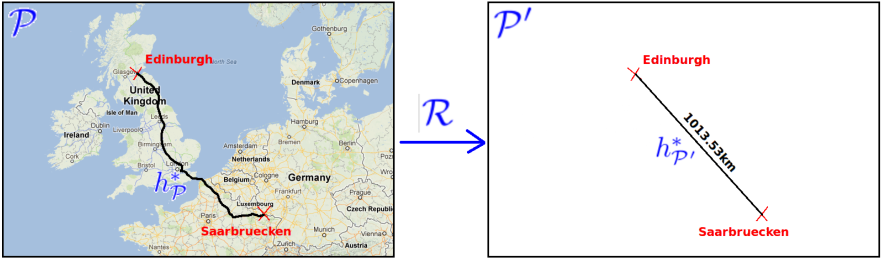
Heuristic Functions from Relaxed Problems

How to Relax



- You have a class $\mathcal{P}$ of problems, whose perfect heuristic $h^*_{\mathcal{P}}$ you wish to estimate.
- You define a class $\mathcal{P}'$ of *simpler problems*, whose perfect heuristic $h^*_{\mathcal{P}'}$ can be used to *estimate* $h^*_{\mathcal{P}}$.
- You define a transformation – the relaxation mapping $\mathcal{R}$ – that maps instances $\Pi \in \mathcal{P}$ into instances $\Pi' \in \mathcal{P}'$.
- Given $\Pi \in \mathcal{P}$, you let $\Pi' := \mathcal{R}(\Pi)$, and estimate $h^*_{\mathcal{P}}(\Pi)$ by $h^*_{\mathcal{P}'}(\Pi')$.

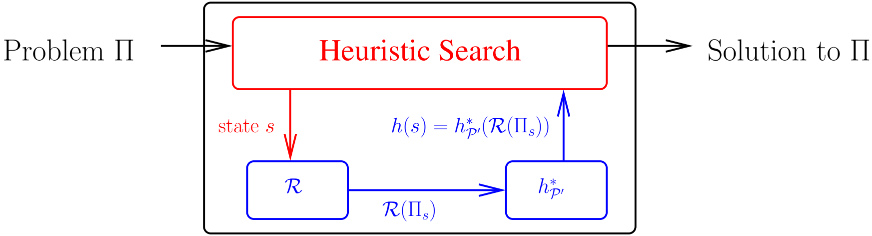
Relaxation in Route-Finding



- Problem class $\mathcal{P}$: Route finding.
- Perfect heuristic $h^*_{\mathcal{P}}$ for $\mathcal{P}$: Length of a shortest route.
- Simpler problem class $\mathcal{P}'$:
- Perfect heuristic $h^*_{\mathcal{P}'}$ for $\mathcal{P}'$:
- Transformation $\mathcal{R}$:

How to Relax During Search: Overview

Attention! Search uses the real (un-relaxed) Π. The relaxation is applied **only within the call to** $h(s)$**!!!**

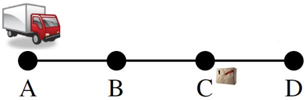


- Here, Π_s is Π with initial state replaced by s, i.e., $\Pi = (P, A, I, G)$ changed to (P, A, s, G): The task of finding a plan for search state s.
- A common student mistake is to instead apply the relaxation once to the whole problem, then doing the whole search "within the relaxation".
- Slides 34 and 32 illustrate the correct search process in detail.

How to Relax During Search: Ignoring Deletes

A Simple Planning Relaxation: Only-Adds

Example: "Logistics"



- Facts P: $\{truck(x) \mid x \in \{A, B, C, D\}\} \cup$ $pack(x) \mid x \in \{A, B, C, D, T\}\}$.
- Initial state I: $\{truck(A)$, $pack(C)\}$.
- Goal G: $\{truck(A)$, $pack(D)\}$.
- Actions A: (Notated as "precondition $\Rightarrow$ adds, $\neg$ deletes")
  - $drive(x, y)$, where x, y have a road: "$truck(x) \Rightarrow truck(y), \neg truck(x)$".
  - $load(x)$: "$truck(x), pack(x) \Rightarrow pack(T), \neg pack(x)$".
  - $unload(x)$: "$truck(x), pack(T) \Rightarrow pack(x), \neg pack(T)$".

Only-Adds Relaxation: Drop the preconditions and deletes.

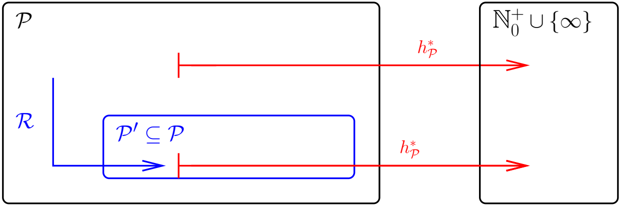
"$drive(x, y)$: $\Rightarrow truck(y)$"; "$load(x)$: $\Rightarrow pack(T)$"; "$unload(x)$: $\Rightarrow pack(x)$".

$\rightarrow$ Heuristic value for I is?

How to Relax During Search: Only-Adds

Only-Adds and Ignoring Deletes are "Native" Relaxations

Native Relaxations: Confusing special case where $\mathcal{P}' \subseteq \mathcal{P}$.



- Problem class $\mathcal{P}$: STRIPS planning tasks.
- Perfect heuristic $h^*_{\mathcal{P}}$ for $\mathcal{P}$: Length h^* of a shortest plan.
- Transformation $\mathcal{R}$: Drop the (preconditions and) delete lists.
- Simpler problem class $\mathcal{P}'$ is a special case of $\mathcal{P}$, $\mathcal{P}' \subseteq P$: STRIPS planning tasks with empty (preconditions and) delete lists.
- Perfect heuristic for $\mathcal{P}'$: Shortest plan for only-adds respectively delete-free STRIPS task.

Questionnaire

Question!

Is Only-Adds a "good heuristic" (accurate goal distance estimates) in ...

(A): Path Planning?

(B): Blocksworld?

(C): Freecell?

(D): SAT? (#unsatisfied clauses)

Summary

- Heuristic functions h map states to estimates of remaining cost. A heuristic can be safe, goal-aware, admissible, and/or consistent. A heuristic may dominate another heuristic, and an ensemble of heuristics may be additive.
- Greedy best-first search can be used for satisficing planning, A^* can be used for optimal planning provided h is admissible. Weighted A^* interpolates between the two.
- Relaxation is a method to compute heuristic functions. Given a problem $\mathcal{P}$ we want to solve, we define a relaxed problem $\mathcal{P}'$. We derive the heuristic by mapping into $\mathcal{P}'$ and taking the solution to this simpler problem as the heuristic estimate.
- During search, the relaxation is used *only inside* the computation of $h(s)$ on each state s; the relaxation does not affect anything else.

Reading

- AI'18 Chapters 4 and 5.
- A word of caution regarding *Artificial Intelligence: A Modern Approach (Third Edition)* [Russell and Norvig (2010)], Sections 3.6.2 and 3.6.3.

  Content: These little sections are aimed at describing basically what I call "How to Relax" here. They do serve to get some intuitions. However, strictly speaking, they're a bit misleading. Formally, a pattern database (Section 3.6.3) *is* what is called a "relaxation" in Section 3.6.2: as we shall see in → **Chapters 11, 12**, pattern databases are abstract transition systems that have more transitions than the original state space. On the other hand, not every relaxation can be usefully described this way; e.g., critical-path heuristics (→ **Chapter 8**) and ignoring-deletes heuristics (→ **Chapter 9**) are associated with very different state spaces.

References I

Jörg Hoffmann and Bernhard Nebel. The FF planning system: Fast plan generation through heuristic search. *Journal of Artificial Intelligence Research*, 14:253–302, 2001.

Robert C. Holte. Common misconceptions concerning heuristic search. In Ariel Felner and Nathan R. Sturtevant, editors, *Proceedings of the 3rd Annual Symposium on Combinatorial Search (SOCS'10)*, pages 46–51, Stone Mountain, Atlanta, GA, July 2010. AAAI Press.

Stuart Russell and Peter Norvig. *Artificial Intelligence: A Modern Approach (Third Edition)*. Prentice-Hall, Englewood Cliffs, NJ, 2010.